iMagX geometry

Rudi Labarbe, Sébastien Brousmiche

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1 Context

The purpose of this document is to describe the geometry format used in iMagX. It is based on the international standard IEC 61217 (Radiotherapy equipments).

2 Modeling the projection of a point on a flat panel

Figure 1 represents the geometrical layout of the rotating X-ray tube and flat panel. The radiographic system is a transformation of the FRS (Fixed Reference System) rotating during the gantry rotation. The gantry rotates by an angle α around the y_{FRS} axis. The origin of the FRS is the point O. In an ideal gantry with no deformations, we have:

- The X-ray tube would be located at a point X. The distance SID (source-to-axis distance) is the distance OX.
- The center of the flat panel would be located at the position F. The distance AID (axis-to-imager distance) is the distance FO. It is thus a negative distance. The sum AID+SID is known as the source-to-detector distance, noted SDD.

In this case, both the tube and the flat panel describes a circular trajectory centered at the isocenter with respective radius SID and AID. In the real world however the gantry undergoes some deformations and therefore:

- The X-ray tube is located at the point $X^{'}$ instead of X. The translations of the tube are denoted $\{S_x, S_y, S_z\}$.
- The center of the flat panel is located at point $F^{'}$ instead of F. In addition, the flat panel undergoes rotations around its axis so that the surface of the detector is not necessarily orthogonal to the OF vector. The translation of the flat panel is denoted $\{D_x, D_y, D_z\}$ and its rotations are: β_x (pitch), β_y (roll) and β_z (yaw).

2.1 The radiographic coordinate system of the tube

Let's define the radiographic coordinate system $\{x_{\text{RAD}}, y_{\text{RAD}}, z_{\text{RAD}}\}$ attached to the X-ray tube. This system can be derived from the FRS coordinate system by the following sequence of transformations:

1. Rotate the FRS coordinate system by an angle α around the y_{FRS} axis.
2. Translate the origin of the intermediate coordinate system (CS) resulting from the last transformation by a distance $SID + S_z$ along a direction parallel to the z_{RAD} axis. The distance is chosen so that the point $X^{'}$ is located in the plane $\{x_{\text{RAD}}, y_{\text{RAD}}\}$. This defines a new intermediate CS.
3. Translate the origin of this new CS by a distance $\{S_x, S_y\}$ in the $\{x_{\text{RAD}}, y_{\text{RAD}}\}$ plane so that the origin of the radiographic coordinate system is located at point $X^{'}$.

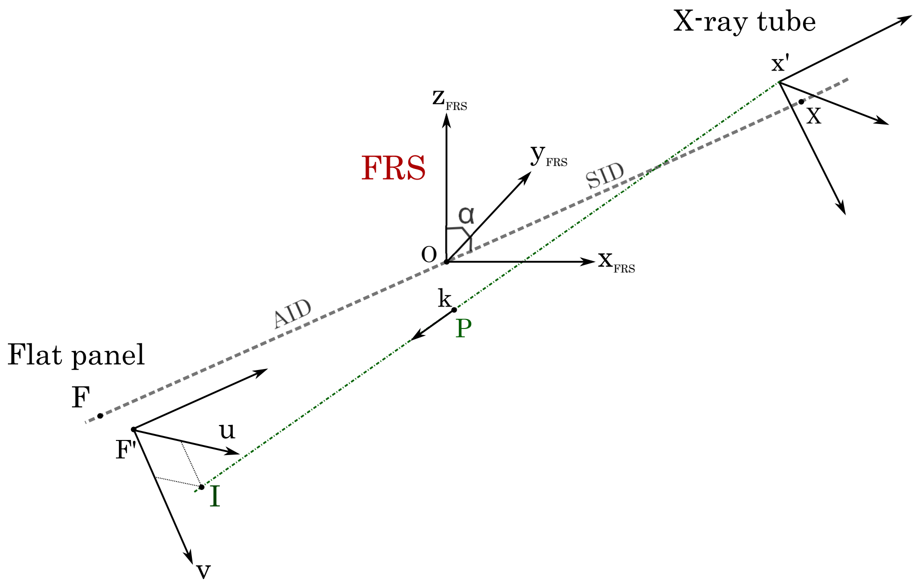

Figure 1: The point O is the origin of the FRS (isocenter). The center of the flat panel is located at the position $F^{'}$ and the source at $X^{'}$ if all deformations are taken into account. Distances SID and SDD = SID+AID are computed without deformations. The projection of a point P on the flat panel is noted I.

The transformation M_t from the FRS to the radiographic CS can then be represented by a product of 4x4 matrices:

$$M_t(\alpha, S_x, S_y, SID + S_z) \quad = \quad R_y(\alpha) \,.\, T_t \tag{1}$$

where

- $R_y(\alpha)$ is a 4x4 matrix representing the rotation around the y_{FRS} axis. It is given by

$$R_y(\alpha) \quad = \quad \begin{pmatrix} \cos(\alpha) & 0 & \sin(\alpha) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

- T_t is a 4x4 translation matrix given by

$$T_t \quad = \quad \begin{pmatrix} 1 & 0 & 0 & S_x \\ 0 & 1 & 0 & S_y \\ 0 & 0 & 1 & SID + S_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

The coordinate of the point $X^{'}$ in the radiographic coordinate system are $\{0, 0, 0\}$ as the point $X^{'}$ is the origin of the coordinate system. The coordinate of a point $X_{\text{RAD}} = \{x, y, z\}_{\text{RAD}}$ expressed in the FRS and denoted X_{FRS} is equal to:

$$OX_{\text{FRS}} \quad = \quad M_x(\alpha, S_x, S_y, SID + S_z) \,.\, OX_{\text{RAD}} \tag{4}$$

2.2 The radiographic coordinate system of the flat panel

Let's define the radiographic coordinate system attached to the flat panel. As for the tube coordinate system, this CS can be derived from the FRS by the following sequence of transformations:

1. Rotate the FRS coordinate system by an angle α around the y_{FRS} axis.
2. Rotate the resulting CS by an angle β_z around the z_{FRS} axis. This angle defines the in-plane rotation of the flat panel.
3. Translate the origin of the previous CS by a distance $AID + D_z$ along a direction parallel to the z_{RAD} axis.
4. Translate the origin of this new CS by a distance $\{D_x, D_y\}$ in the $\{x_{\text{RAD}}, y_{\text{RAD}}\}$ plane so that the origin of the radiographic coordinate system is located at point $F^{'}$.
5. Rotate the resulting CS by an angle β_y around the y_{RAD} axis.
6. Rotate the resulting CS by an angle β_x around the x_{RAD} axis.

The transformation M_d from the FRS to the radiographic CS can then be represented by a product of 4x4 matrices:

$$M_d(\alpha, D_x, D_y, AID + D_z, \beta_x, \beta_y, \beta_z) \quad = \quad R_y(\alpha)\ .\ R_z(\beta_z)\ .\ T_d\ .\ R_y(\beta_y)\ .\ R_x(\beta_x) \tag{5}$$

where

- R_i are 4x4 rotation matrices around the i_{RAD} axis
- T_d is a 4x4 translation matrix given by

$$T_d \quad = \quad \begin{pmatrix} 1 & 0 & 0 & D_x \\ 0 & 1 & 0 & D_y \\ 0 & 0 & 1 & AID + D_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

Let's now compute the components of the $\vec{u}$ and $\vec{v}$ vectors expressed in the FRS and defining the radiographic CS attached to the flat panel. The $\vec{u}$ defines the x_{RAD} axis and the $\vec{v}$ defines the y_{RAD} axis. The $\vec{u}$ vector is defined by the two points with coordinates $\{0, 0, 0\}_{\text{RAD}}$ and $\{1, 0, 0\}_{\text{RAD}}$ expressed in the radiographic CS. We can therefore define the components $\vec{u}_{\text{FRS}} = \{x_u, y_u, z_u\}_{\text{FRS}}$ of the vector in the FRS using:

$$\vec{u}_{\text{FRS}} \quad = \quad M_d \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}_{\text{RAD}} - M_d \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}_{\text{RAD}} \tag{7}$$

Similarly, the $\vec{v}$ vector is defined by the two points with coordinates $\{0, 0, 0\}_{\text{RAD}}$ and $\{0, 1, 0\}_{\text{RAD}}$ expressed in the radiographic CS. We can therefore define the components $\vec{v}_{\text{FRS}} = \{x_v, y_v, z_v\}_{\text{FRS}}$ of the vector in the FRS using:

$$\vec{v}_{\text{FRS}} \quad = \quad M_d \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}_{\text{RAD}} - M_d \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}_{\text{RAD}} \tag{8}$$

2.3 Projection of a point on the flat panel

Let's define a point P contained in an object placed near the origin of the FRS (isocenter). The coordinates of the point P in the FRS are $\{x_{\text{P}}, y_{\text{P}}, z_{\text{P}}\}_{\text{FRS}}$. The X-ray source will project this point P onto the flat panel at the position I with coordinates $\{x_{\text{I}}, y_{\text{I}}\}_{\text{RAD}}$ in the $(\vec{u}, \vec{v})$ system.

The points $X^{'}$ (defining the actual position of the source), P and I are located on the same line with direction vector $\vec{t}$. One can write:

$$\vec{OF'} + \vec{F'I} = \vec{OP} + \vec{PI} \tag{9}$$

with

$$\vec{F'I} = x_{\text{I}}\ \vec{u} + y_{\text{I}}\ \vec{v} \tag{10}$$

and

$$\vec{PI} = k\ \vec{t} \tag{11}$$

where k is a parameter which is still unknown. Using Equations 9, 10 and 11, we find:

$$\vec{OP} - \vec{OF'} = x_{\text{I}}\ \vec{u} + y_{\text{I}}\ \vec{v} - k\ \vec{t} \tag{12}$$

If we express this last equation in coordinates instead of vector, we get the following system of equations:

$$\left\{ \begin{array}{rcl} x_{\text{P}} - x_{\text{F'}} & = & x_{\text{I}} x_u + y_{\text{I}} x_v - k x_t \\ y_{\text{P}} - y_{\text{F'}} & = & x_{\text{I}} y_u + y_{\text{I}} y_v - k y_t \\ z_{\text{P}} - z_{\text{F'}} & = & x_{\text{I}} z_u + y_{\text{I}} z_v - k z_t \end{array} \right. \tag{13}$$

In this equation, the only unknowns are the coordinates x_{I}, y_{I} and the coefficient k. We can solve the previous system of equation using the Kramers's determinants:

$$x_{\text{I}} = D^{-1} \begin{vmatrix} x_{\text{P}} - x_{\text{F'}} & x_v & -x_t \\ y_{\text{P}} - y_{\text{F'}} & y_v & -y_t \\ z_{\text{P}} - z_{\text{F'}} & z_v & -z_t \end{vmatrix} \tag{14}$$

$$y_{\text{I}} = D^{-1} \begin{vmatrix} x_u & x_{\text{P}} - x_{\text{F'}} & -x_t \\ y_u & y_{\text{P}} - y_{\text{F'}} & -y_t \\ z_u & z_{\text{P}} - z_{\text{F'}} & -z_t \end{vmatrix} \tag{15}$$

$$k = D^{-1} \begin{vmatrix} x_u & x_v & x_{\text{P}} - x_{\text{F'}} \\ y_u & y_v & y_{\text{P}} - y_{\text{F'}} \\ z_u & z_v & z_{\text{P}} - z_{\text{F'}} \end{vmatrix} \tag{16}$$

where D is given by

$$D = \begin{vmatrix} x_u & x_v & -x_t \\ y_u & y_v & -y_t \\ z_u & z_v & -z_t \end{vmatrix} \tag{17}$$